THE COOPERS' COMPANY AND COBORN SCHOOL

Love as Brethren

A-Level Mathematics Summer Task

(compulsory for all maths students and all further maths students)

Name: _____

Time: 2-5 hours work

Due: First maths lesson in September

Total: _____ out of 200 marks

This summer task provides revision for some of the topics covered in GCSE mathematics that are needed for A Level mathematics.

All students are expected to **answer all questions**, with relevant written calculations. Students may wish to attempt all questions independently first, then use their notes or other resources to help with more difficult questions.

You may use a **calculator** for all questions unless otherwise indicated.

This work will be handed in during the **first mathematics lesson in September**.

The Mathematics department look forward to seeing you in September.

Q1.

$x_{n+1} = \sqrt[3]{3x_n + 7}$

Use a starting value of $x_1 = 2$ to work out a solution to $x = \sqrt[3]{3x + 7}$

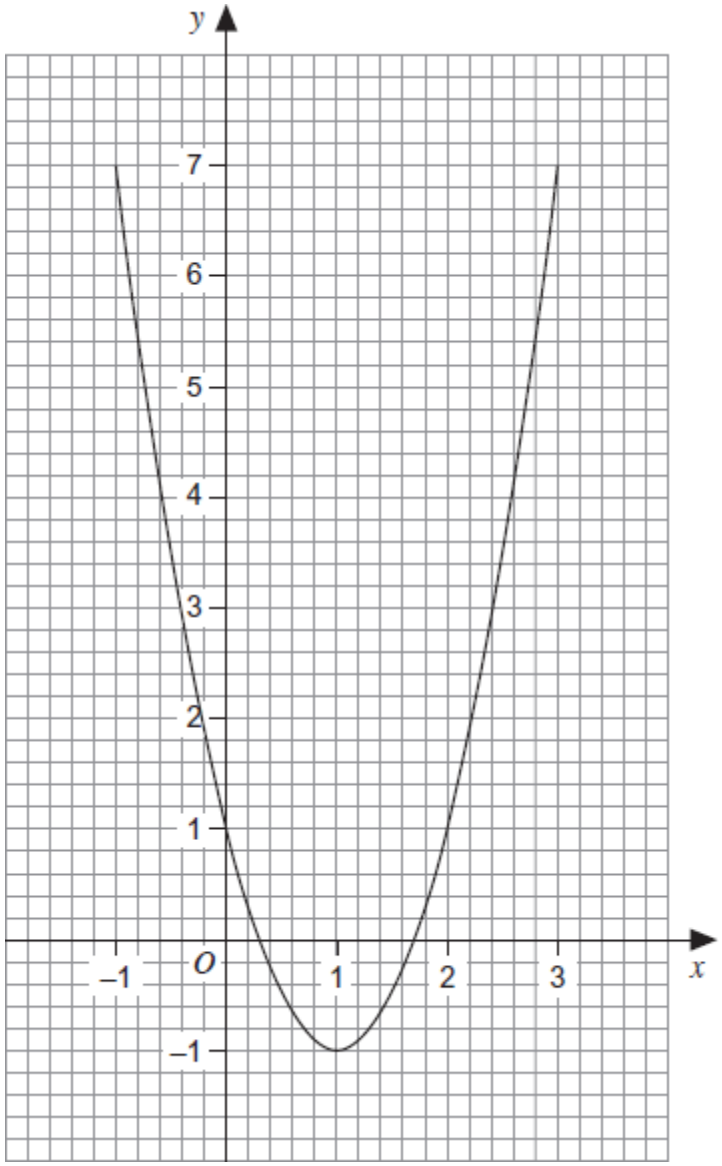
Give your answer to 3 decimal places.

Answer _____

(Total 3 marks)

Q2.

Here is the graph of $y = 2x^2 - 4x + 1$ for values of x from −1 to 3



Use the graph to estimate the solutions to $2x^2 - 4x + 1 = 5$

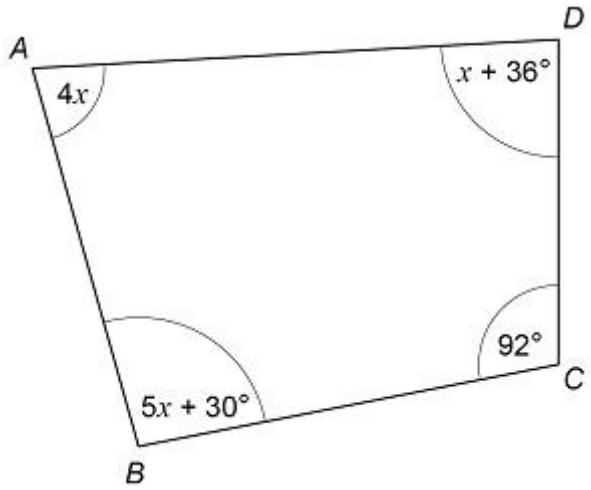
Answer _____

(Total 2 marks)

Q3.

ABCD is a quadrilateral.

Not drawn accurately



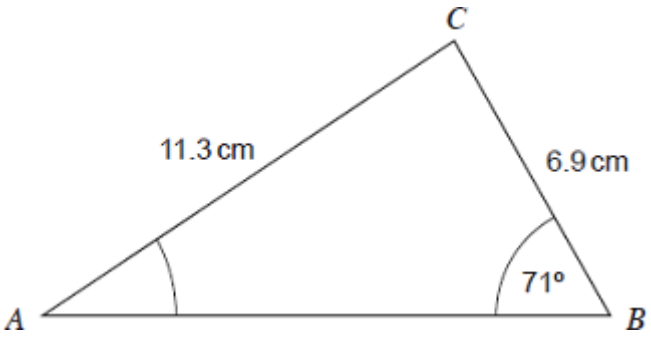
Prove that *ABCD* is **not** a cyclic quadrilateral.

(Total 4 marks)

Q4.

Work out the size of angle A.

Not drawn accurately

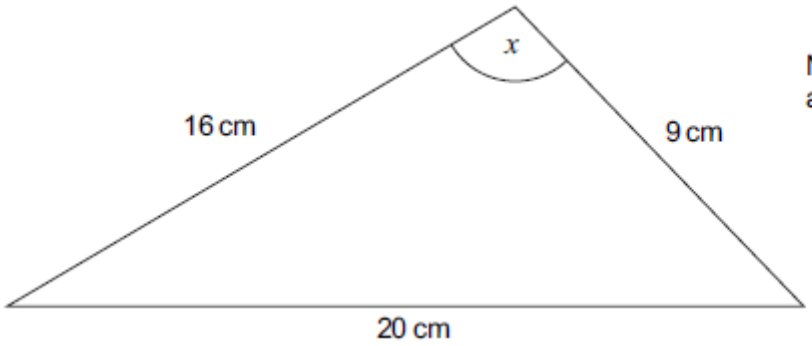


Give your answer to a suitable degree of accuracy.

Answer _____ degrees

(Total 4 marks)

Q5.



Work out angle x.

_____

_____

_____

_____

_____

_____

x = _____ degrees

(Total 3 marks)

Q6.

The graph of $y = x^2 + 2x - 3$ is drawn below.

Draw an appropriate **straight** line on the graph to work out the approximate solutions of

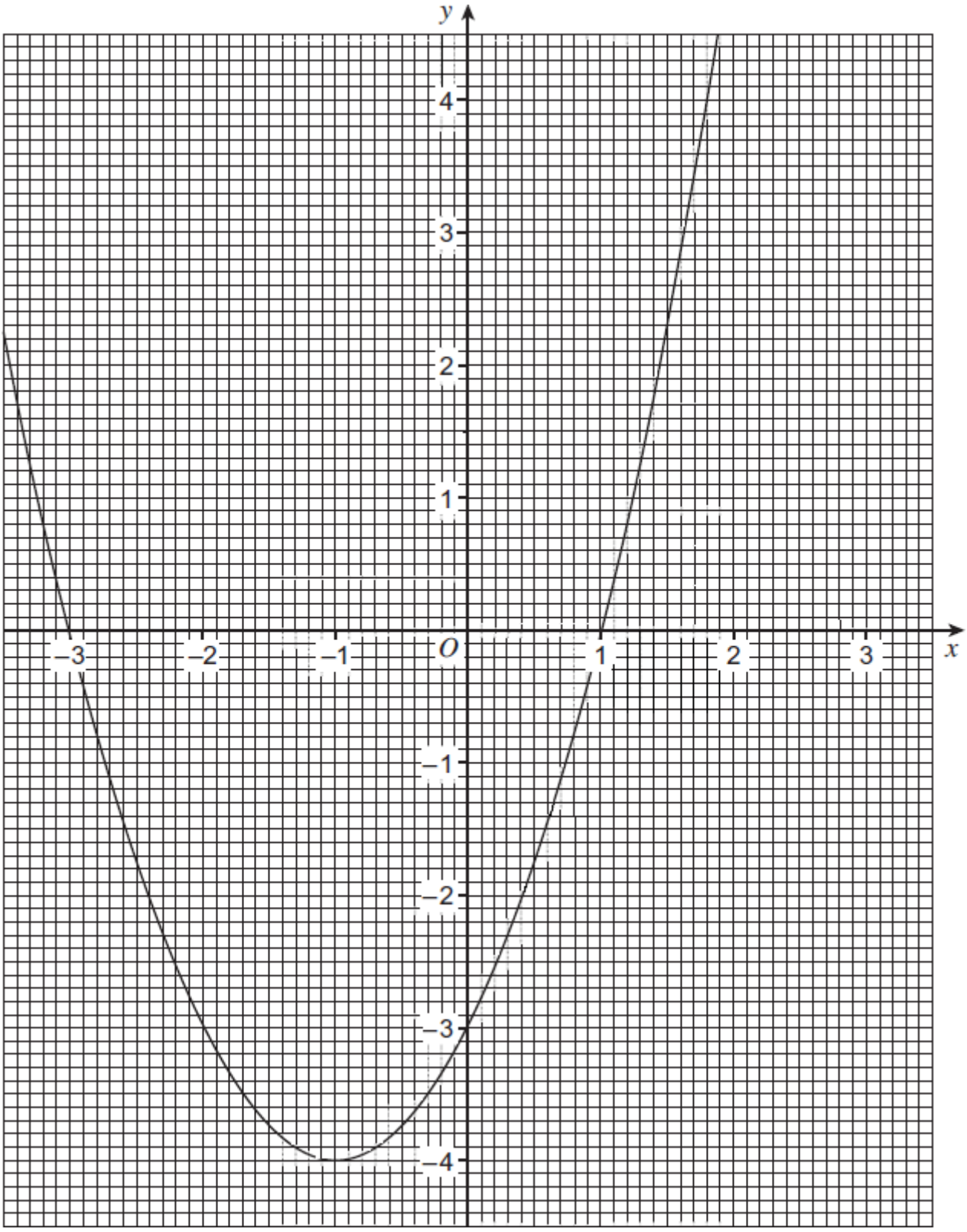
$$x^2 + x - 3 = 0$$

_____

_____

Answer _____

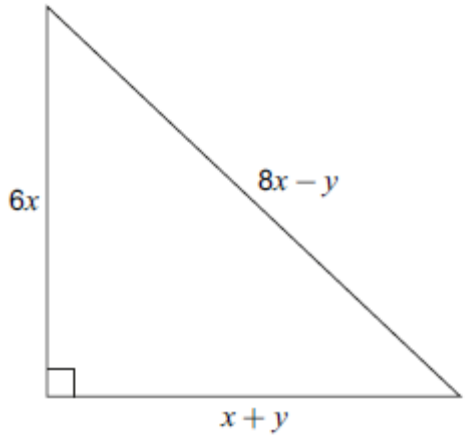
$y = x^2 + 2x - 3$



(Total 3 marks)

Q7.

The diagram shows a right-angled triangle.



Not drawn accurately

Prove algebraically that $x : y = 2 : 3$

(Total 6 marks)

Q8.

Solve the simultaneous equations

$$y = x^2 - 6x - 20$$

$$y = 4 - x$$

You **must** show your working.

Answer _____

(Total 5 marks)

Q9.

Solve the quadratic equation $5x^2 + 8x + 2 = 0$

Give your answers to 1 decimal place.

Answer _____

(Total 3 marks)

Q10.

$(3x + 1)(x - 2) + ax + b \equiv 3x^2 + 8x - 5$

Work out the values of a and b.

a =.....

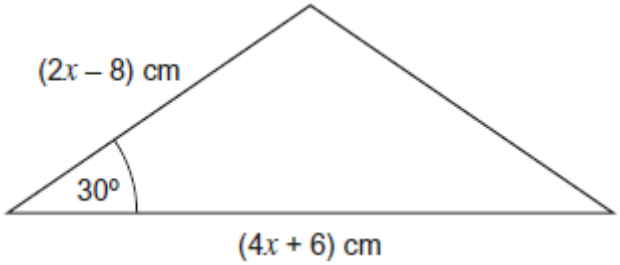
b =.....

(Total 4 marks)

Q11.

The area of this triangle is 14 cm^2

Not drawn accurately



(a) Show that $2x^2 - 5x - 26 = 0$

(3)

(b) Work out the value of x.
Give your answer to 2 significant figures.

Answer _____

(4)

(Total 7 marks)

Q12.

Curve P has equation $y = 2(x - 1)^2 - 5$

Curve Q is a reflection in the y-axis of curve P.

Work out the equation of curve Q.

Give your answer in the form $y = ax^2 + bx + c$ where a, b and c are integers.

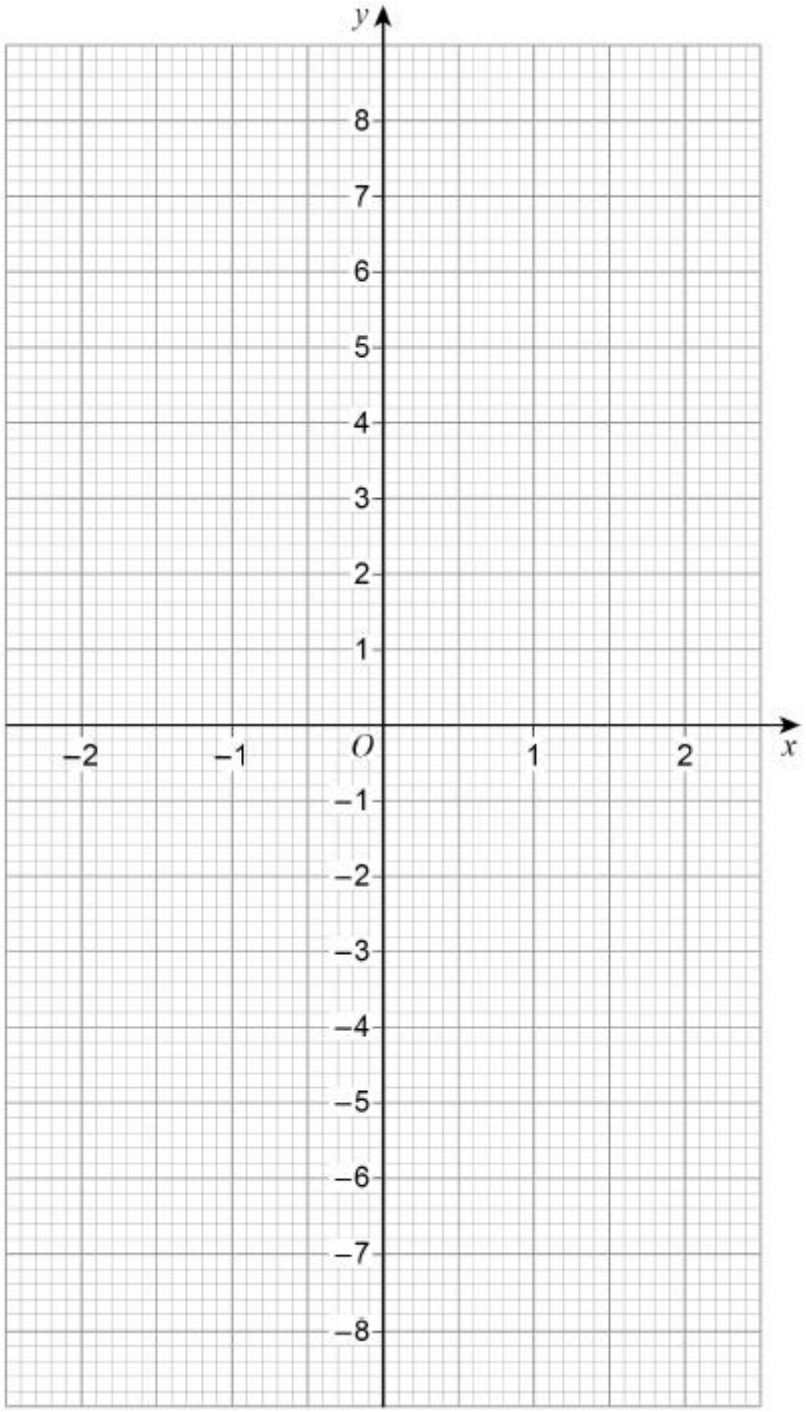
Answer _____

(Total 3 marks)

Q13.

(a) $h(x) = \sqrt[3]{x}$ for all values of x

On the grid, draw the graph of the inverse function $y = h^{-1}(x)$ for $-2 \leq x \leq 2$



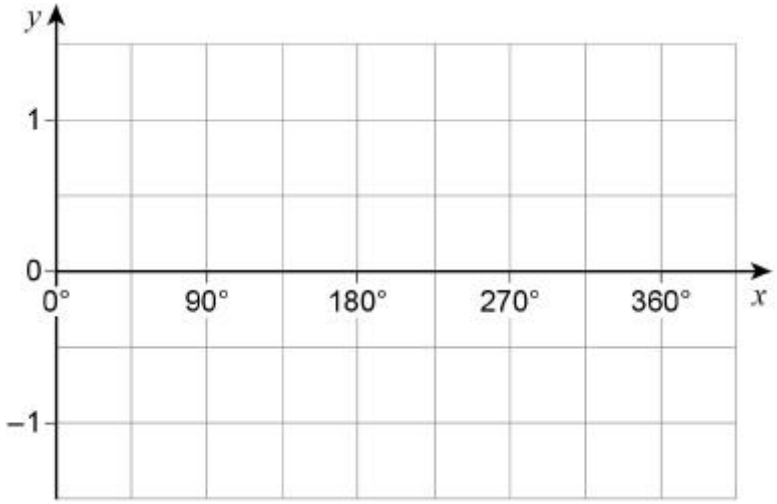
(2)

(b) For all values of x

$f(x) = \sin x$

$g(x) = x + 90$

On the grid, draw the graph of the composite function $y = fg(x)$ for $0° \leq x \leq 360°$



(2)

(Total 4 marks)

Q14.

$f(x) = \frac{1}{2}x$ $g(x) = x - x^2$

Solve $f^{-1}(x) = gf(x)$

Answer _____

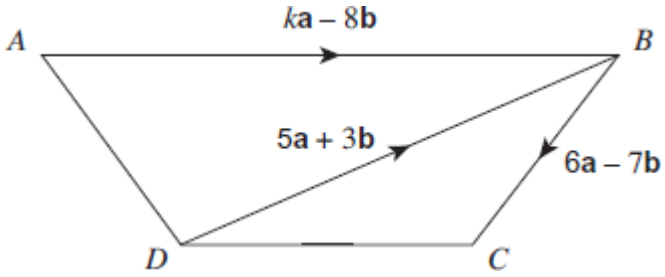
(Total 4 marks)

Q15.

Prove that $3(x + 1)(x + 7) - (2x + 5)^2$ is never positive.

(Total 5 marks)

Q16.



(a) Work out $\overrightarrow{DC}$ in terms of **a** and **b**.
Simplify your answer.

_____

_____

Answer _____

(2)

(b) *ABCD* is a trapezium.

Work out the value of k.

_____

_____

Answer _____

(1)

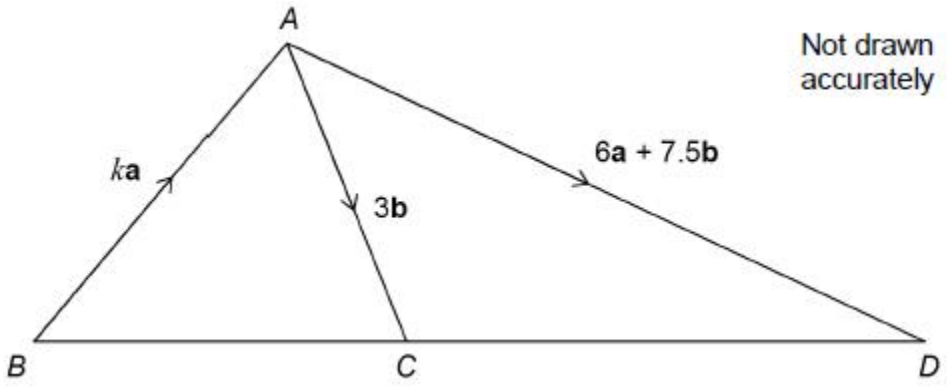
(Total 3 marks)

Q17.

ABC and *ACD* are triangles.

k is a constant.

Not drawn accurately



(a) Show that $\overrightarrow{CD} = 6\mathbf{a} + 4.5\mathbf{b}$

_____

_____

_____

(1)

(b) *BCD* is a straight line.

Work out the value of k.

You **must** show your working.

_____

_____

_____

_____

_____

Answer _____

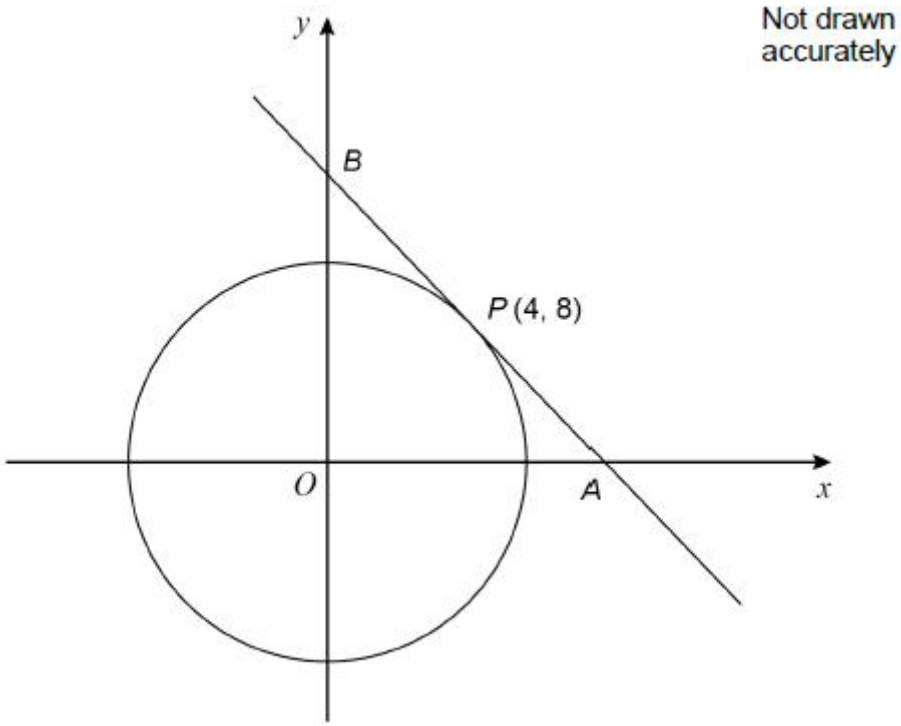
(3)

(Total 4 marks)

Q18.

P (4, 8) is a point on a circle, centre *O*.

The tangent at *P* intersects the axes at points *A* and *B*.



(a) Show that the gradient of the tangent is $-\frac{1}{2}$

(2)

(b) Work out the length *AB*.

Give your answer in the form $a\sqrt{5}$ where a is an integer.

You **must** show your working.

Answer _____ units

(4)

(Total 6 marks)

Q19.

Solve $\frac{5}{4x+1} = \frac{2x}{x^2+3}$

Give your solutions to 3 significant figures.

You **must** show your working.

Answer _____

(Total 5 marks)

Q20.

$f(x) = \dfrac{2x+3}{x-4}$

Work out $f^{-1}(x)$

Answer _____

(Total 4 marks)

Q21.

A school has 86 teachers.

42 are male and 44 are female.

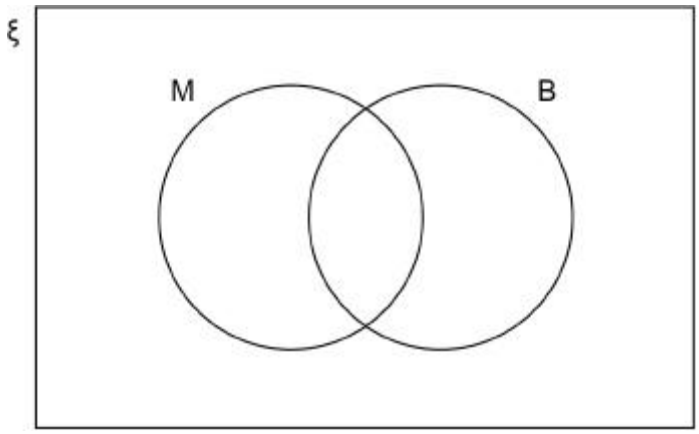
$\frac{1}{3}$ of the male teachers have blue eyes.

$\frac{1}{4}$ of the female teachers have blue eyes.

(a) ξ = teachers in the school

M = male teachers

B = teachers who have blue eyes



Complete the Venn diagram.

_____

_____

_____

(3)

(b) One teacher who has blue eyes is chosen at random.

Work out the probability that the teacher is male.

Answer _____

(1)

(Total 4 marks)

Q22.

A bag contains 30 discs.

10 are red and 20 are blue.

One disc is taken out at random and replaced by **two** of the other colour.
Another disc is then taken out at random and replaced by **two** of the other colour.
Another disc is then taken out at random.

Work out the probability that all three discs taken out are **red**.

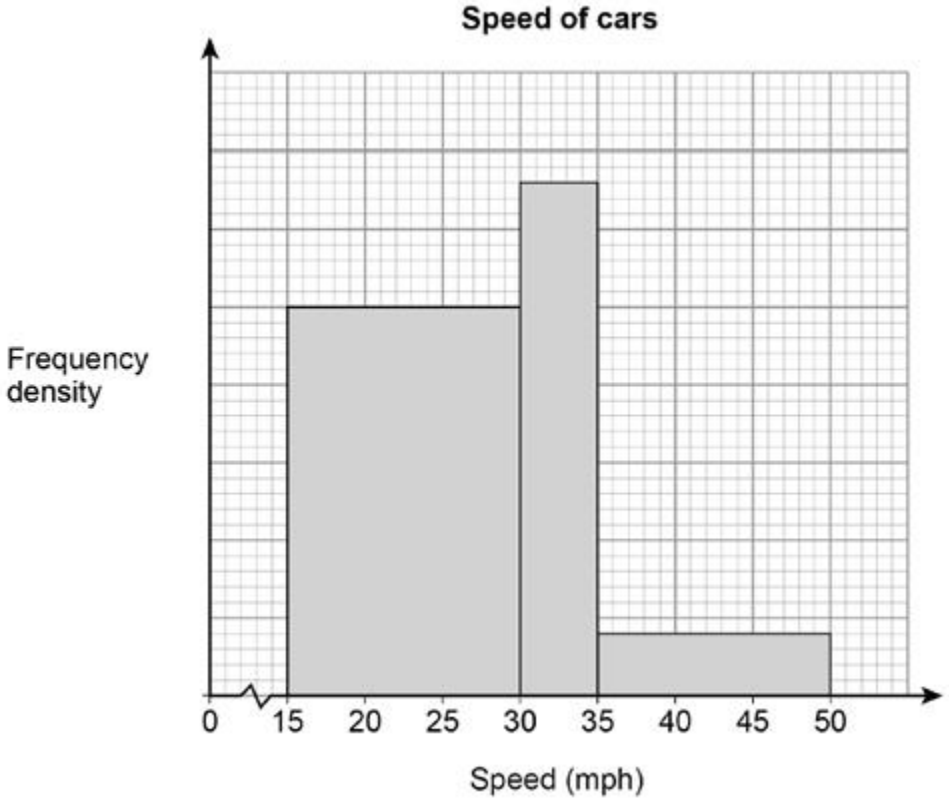
Answer _____

(Total 3 marks)

Q23.

The histogram shows information about the speed of cars as they pass a checkpoint.

The scale on the frequency density axis is missing.



The histogram shows information about 480 cars.

(a) How many cars does the first bar represent?

Answer _____

(4)

(b) Cars with a speed greater than 40 mph are over the speed limit.

Use the histogram to estimate the number of cars that are over the speed limit.

Answer _____

(2)

(Total 6 marks)

Q24.

The probability that Gina goes to the gym on Saturday is 0.9
The probability that Dave goes to the gym on Saturday is 0.6
These probabilities are **independent**.

(a) Calculate the probability that **both** Gina and Dave go to the gym on Saturday.

Answer _____

(1)

(b) If Gina goes to the gym on Saturday the probability that she goes on Sunday is 0.2
If Gina does **not** go to the gym on Saturday the probability that she goes on Sunday is 0.7

Calculate the probability that Gina goes to the gym on exactly **one** of the two days.

Answer _____

(4)

(Total 5 marks)

Q25.

Show that $(x + 1)(x + 2)(x + 3)$ can be written in the form $ax^3 + bx^2 + cx + d$ where a, b, c and d are positive integers.

(Total for question = 3 marks)

Q26.

Prove algebraically that the difference between the squares of any two consecutive odd numbers is always a multiple of 8

(Total for question = 3 marks)

Q27.

Show that $\frac{3+\sqrt{2}}{5+\sqrt{8}}$ can be written as $\frac{11-\sqrt{2}}{17}$

(Total for question = 3 marks)

Q28.

(a) Simplify $\frac{x^2-16}{2x^2-5x-12}$

.....

(3)

(b) Make v the subject of the formula $w = \dfrac{15(t - 2v)}{v}$

.....

(3)

(Total for question = 6 marks)

Q29.

Show that $\dfrac{7x - 14}{x^2 + 4x - 12} \div \dfrac{x - 6}{x^3 - 36x}$ simplifies to ax where a is an integer.

(Total for question = 4 marks)

Q30.

Solve $x^2 > 3x + 4$

.....

(Total for question = 3 marks)

Q31.

Two solid cones are mathematically similar.

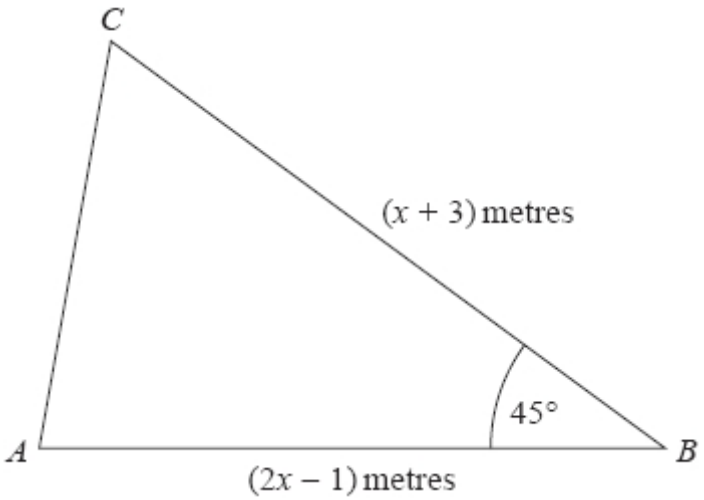
Cone **A** has a volume of 120 cm^3
Cone **B** has a volume of 960 cm^3

Work out the ratio of the surface area of cone **A** to the surface area of cone **B**.

.....

(Total for question = 3 marks)

Q32.



The area of triangle *ABC* is $6\sqrt{2}\,m^2$.

Calculate the value of *x*.
Give your answer correct to 3 significant figures.

.....

(Total for question = 5 marks)

Q33.

L is the circle with equation $x^2 + y^2 = 4$

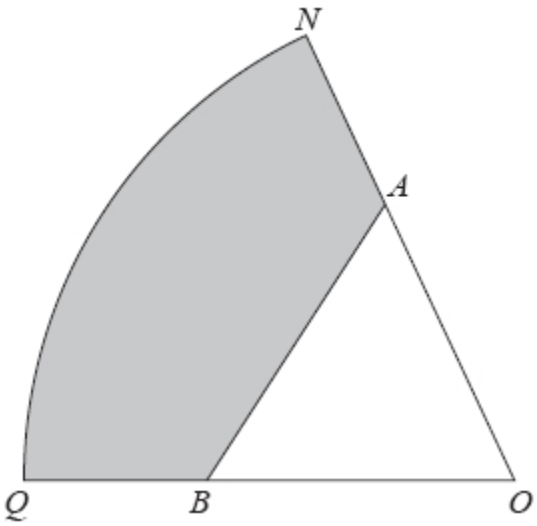
$P\left(\frac{3}{2}, \frac{\sqrt{7}}{2}\right)$ is a point on **L**.

Find an equation of the tangent to **L** at the point *P*.

.....

(Total for question = 3 marks)

Q34.



ONQ is a sector of a circle with centre *O* and radius 11 cm.

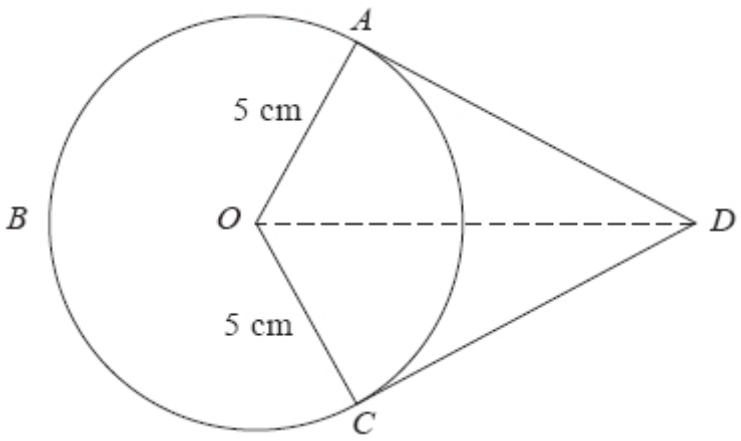
A is the point on *ON* and *B* is the point on *OQ* such that *AOB* is an equilateral triangle of side 7 cm.

Calculate the area of the shaded region as a percentage of the area of the sector *ONQ*.
Give your answer correct to 1 decimal place.

..... %

(Total for question = 5 marks)

Q35.



A, *B* and *C* are points on a circle of radius 5 cm, centre *O*.
DA and *DC* are tangents to the circle.
DO = 9 cm

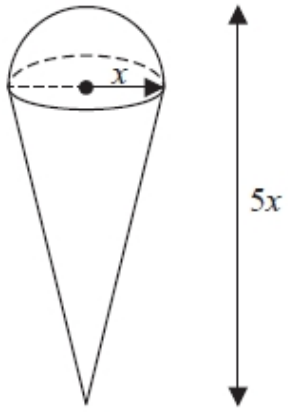
Work out the length of arc *ABC*.
Give your answer correct to 3 significant figures.

..... cm

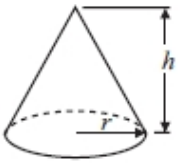
(Total for question = 5 marks)

Q36.

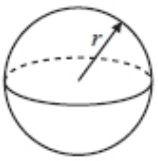
A solid is made by putting a hemisphere on top of a cone.



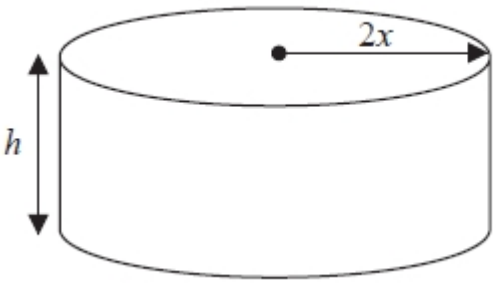
Volume of cone $= \frac{1}{3}\pi r^2 h$



Volume of sphere $= \frac{4}{3}\pi r^3$



The total height of the solid is 5*x*
The radius of the base of the cone is *x*
The radius of the hemisphere is *x*



A cylinder has the same volume as the solid.
The cylinder has radius 2*x* and height *h*
All measurements are in centimetres.

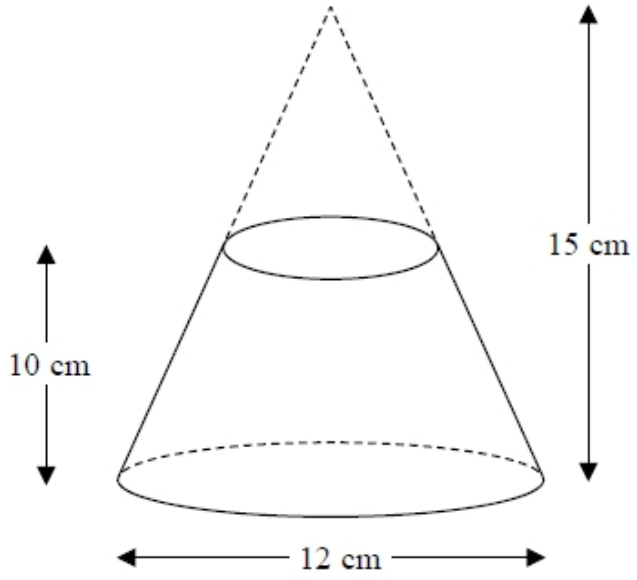
Find a formula for *h* in terms of *x*
Give your answer in its simplest form.

.....

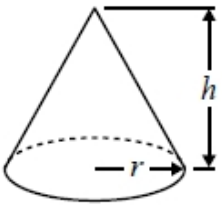
(Total for question = 5 marks)

Q37.

A frustum is made by removing a small cone from a large cone as shown in the diagram.



Volume of cone $= \frac{1}{3}\pi r^2 h$



The frustum is made from glass.
The glass has a density of 2.5 g / cm^3

Work out the mass of the frustum.
Give your answer to an appropriate degree of accuracy.

..... g

(Total for question = 5 marks)

Q38.

The diagram shows a sphere and a solid cylinder.

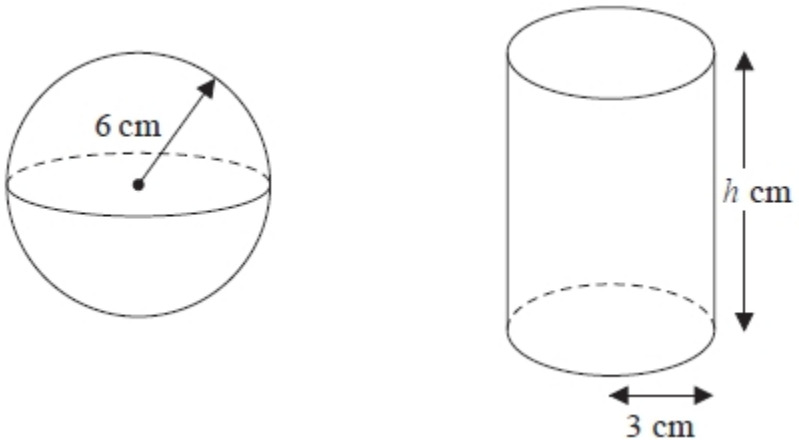


Diagram **NOT** accurately drawn

The sphere has radius 6 cm.
The solid cylinder has a base radius of 3 cm and a height of h cm.

The total surface area of the cylinder is twice the total surface area of the sphere.

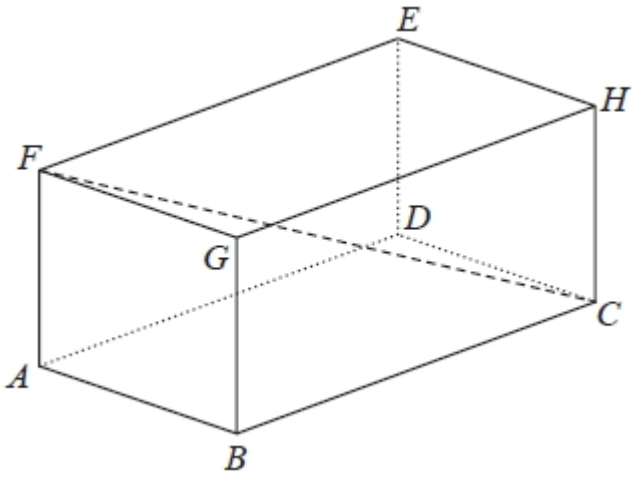
Work out the ratio of the volume of the sphere to the volume of the cylinder.
Give your answer in its simplest form.
You must show all your working.

.....

(Total for question = 5 marks)

Q39.

The diagram shows a cuboid *ABCDEFGH*.



AB = 7 cm, *AF* = 5 cm and *FC* = 15 cm.

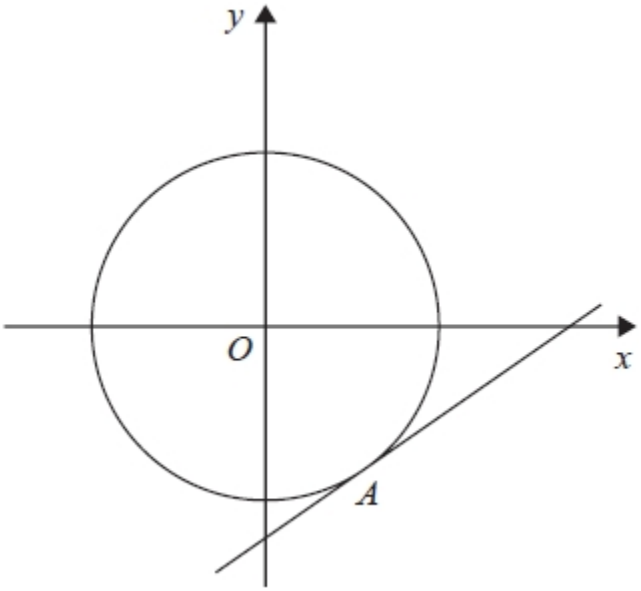
Calculate the volume of the cuboid.
Give your answer correct to 3 significant figures.

..... cm^3

(Total for question is 4 marks)

Q40.

The diagram shows the circle with equation $x^2 + y^2 = 261$



A tangent to the circle is drawn at point A with coordinates $(p, -15)$, where $p > 0$

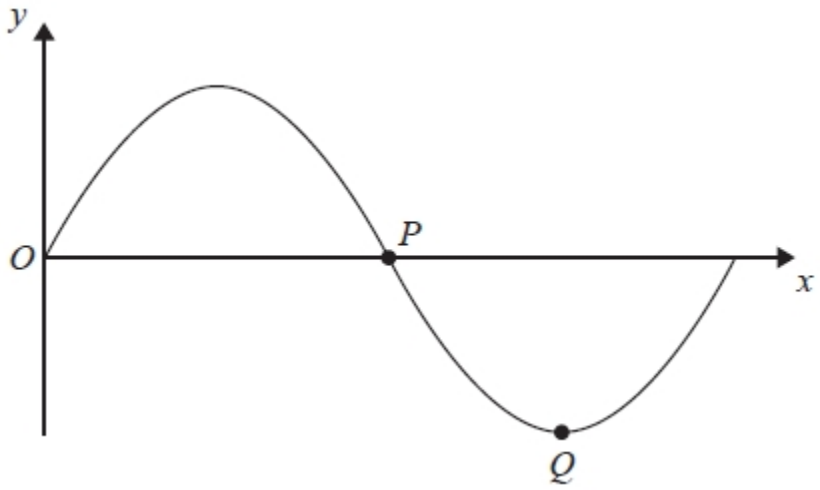
Find an equation of the tangent at A.

.....

(Total for question = 5 marks)

Q41.

The diagram shows part of a sketch of the curve $y = \sin x°$.



(a) Write down the coordinates of the point P.

(..... , ...)

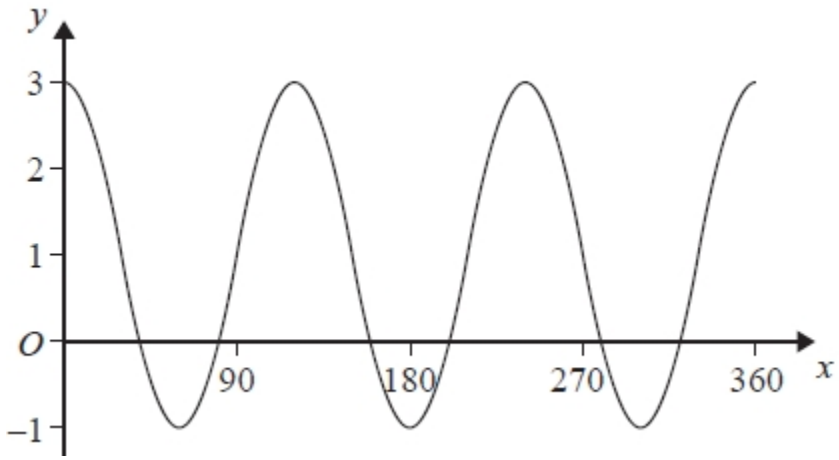
(1)

(b) Write down the coordinates of the point Q.

(..... , ...)

(1)

Here is a sketch of the curve $y = a \cos bx° + c$, $0 \leq x \leq 360$



(c) Find the values of a, b and c.

a =.....

b =.....

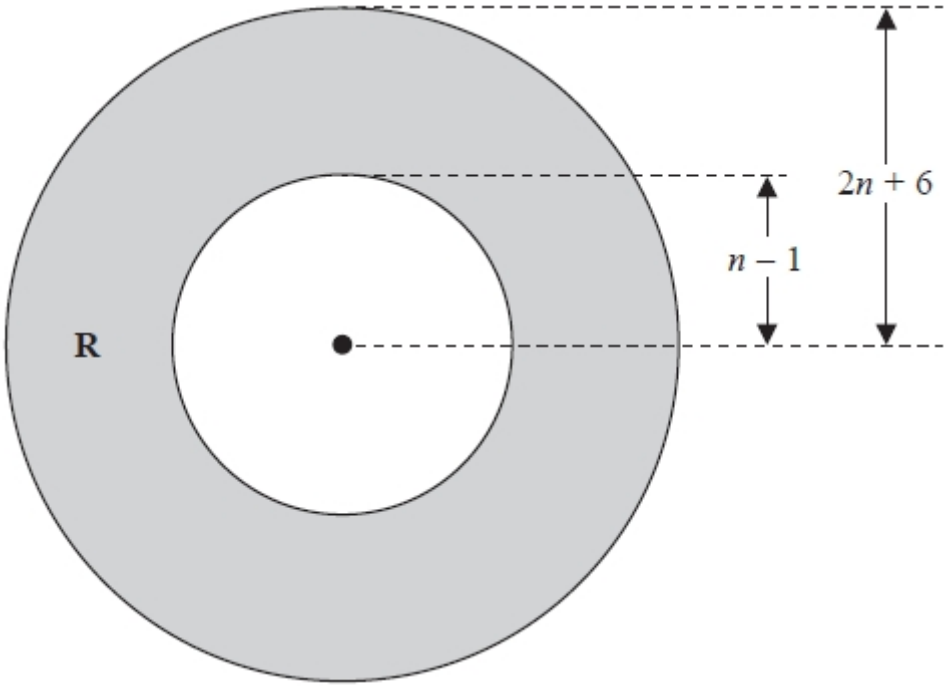
c =.....

(3)

(Total for Question is 5 marks)

Q42.

The region **R**, shown shaded in the diagram, is the region between two circles with the same centre.



The outer circle has radius $(2n + 6)$
The inner circle has radius $(n – 1)$
All measurements are in centimetres.

The area of **R** is greater than the area of a circle of radius $(n + 13)$ cm.

n is an integer.

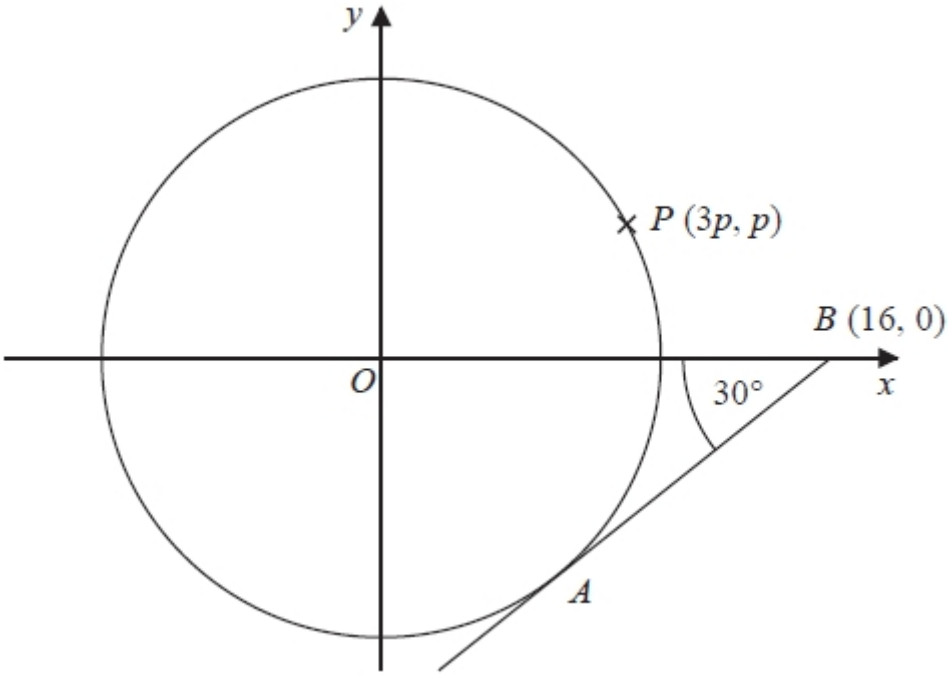
Find the least possible value of n.
You must show all of your working.

.....

(Total for question = 5 marks)

Q43.

The diagram shows a circle, centre *O*.



AB is the tangent to the circle at the point *A*.
Angle *OBA* = 30°

Point *B* has coordinates (16, 0)
Point *P* has coordinates (3*p*, *p*)

Find the value of *p*.
Give your answer correct to 1 decimal place.
You must show all your working.

p = ...

(Total for question = 4 marks)

Q44.

* The diagram shows a triangle *DEF* inside a rectangle *ABCD*.

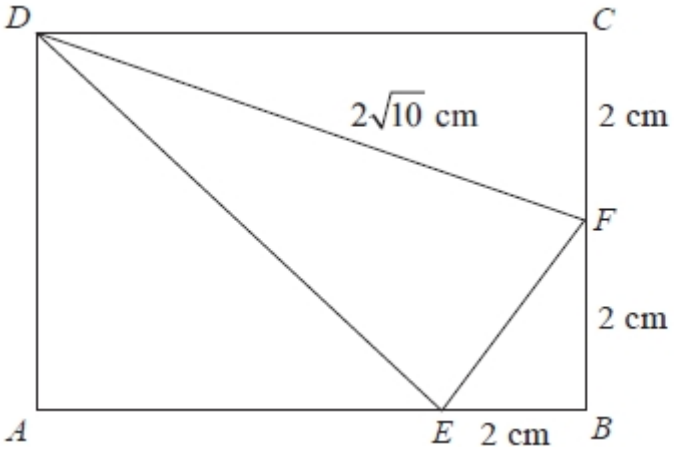


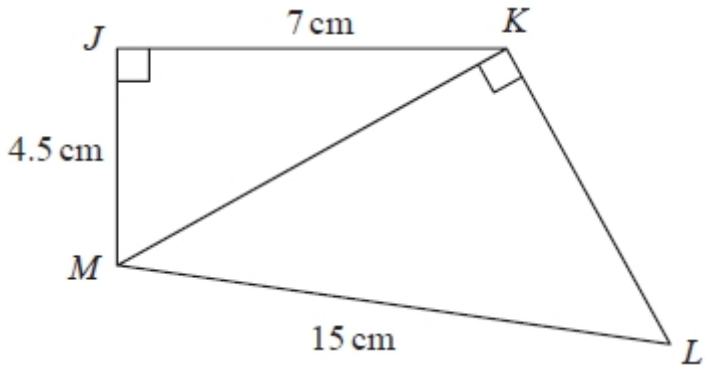
Diagram **NOT** accurately drawn

Show that the area of triangle *DEF* is 8 cm^2.
You must show all your working.

(Total for question = 4 marks)

Q45.

The diagram shows a quadrilateral *JKLM*.



Work out the size of angle *KLM*.
Give your answer correct to 3 significant figures.

..... °

(Total for question = 4 marks)

Q46.

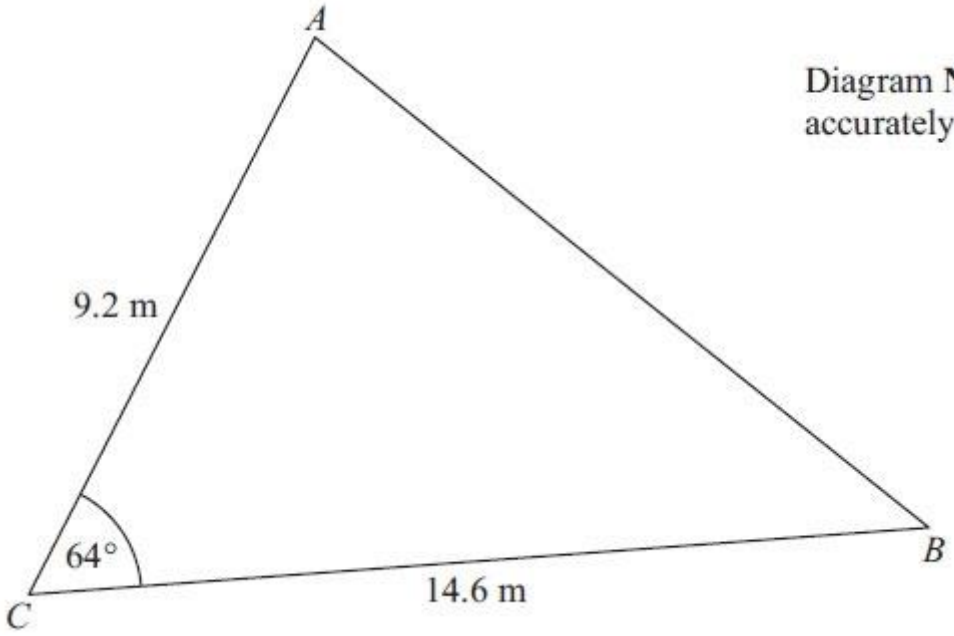


Diagram **NOT** accurately drawn

AC = 9.2 m
BC = 14.6 m
Angle *ACB* = 64°

(a) Calculate the area of the triangle *ABC*.
Give your answer correct to 3 significant figures.

. . . . . m^2
(2)

(b) Calculate the length of *AB*.
Give your answer correct to 3 significant figures.

.....

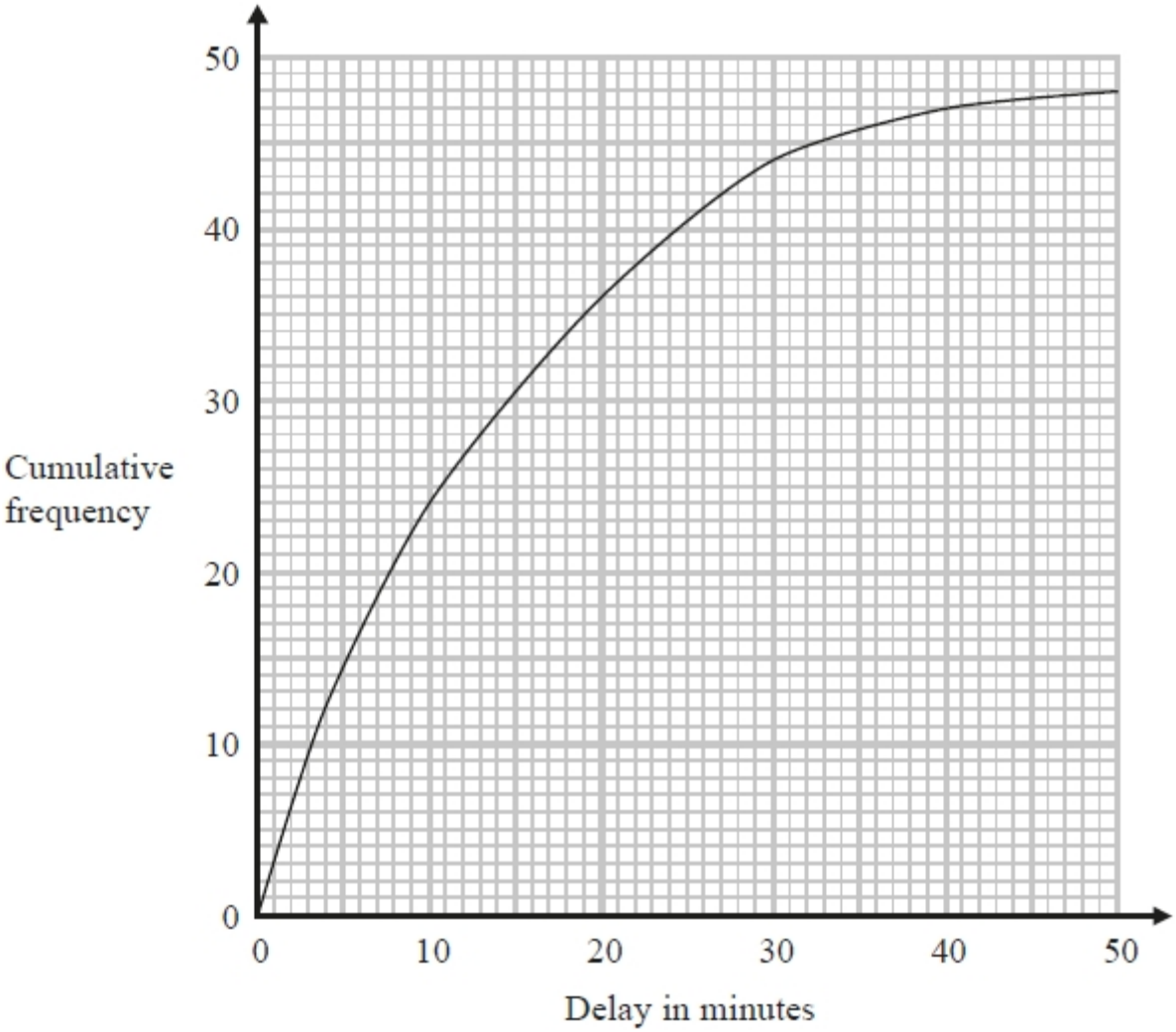
(3)

(Total for Question is 5 marks)

Q47.

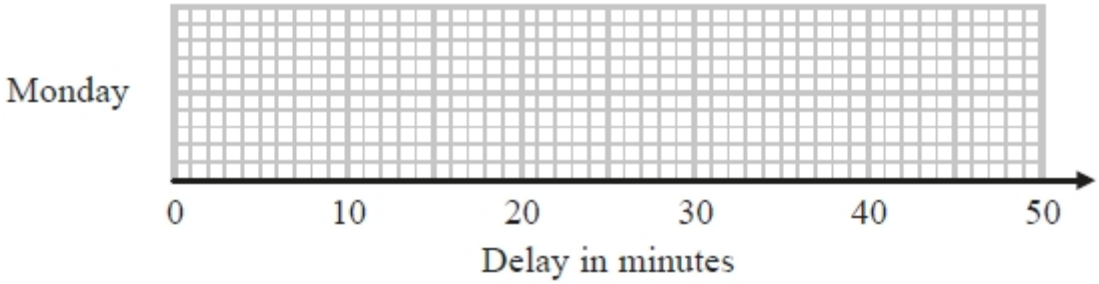
The times that 48 trains left a station on Monday were recorded.

The cumulative frequency graph gives information about the numbers of minutes the trains were delayed, correct to the nearest minute.



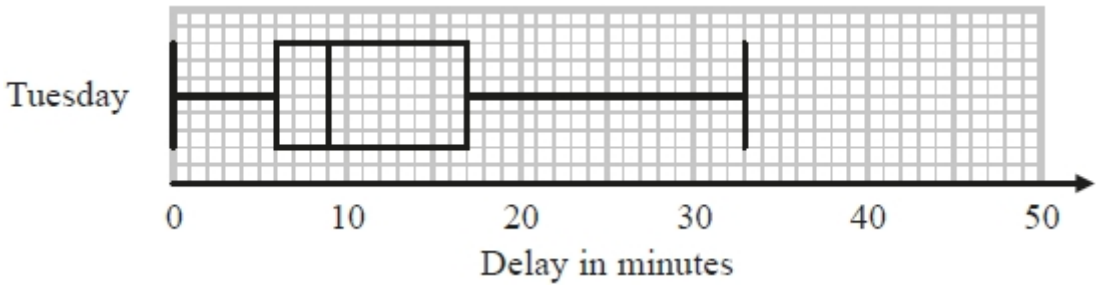
The shortest delay was 0 minutes.
The longest delay was 42 minutes.

(a) On the grid below, draw a box plot for the information about the delays on Monday.



(3)

48 trains left the station on Tuesday.
The box plot below gives information about the delays on Tuesday.



(b) Compare the distribution of the delays on Monday with the distribution of the delays on Tuesday.

.....

.....

.....

.....

(2)

Mary says,

"The longest delay on Tuesday was 33 minutes.
This means that there must be some delays of between 25 minutes and 30 minutes."

(c) Is Mary right?

You must give a reason for your answer.

.....

.....

(1)

(Total for question = 6 marks)