Determining uncertainty

Specification references

- *2.2.1 c)*
- *M0.3 Use ratios, fractions, and percentages*
- *M1.5 Identify uncertainties in measurements and use simple techniques to determine uncertainty when data are combined by addition, subtraction, multiplication, division, and raising to powers*

Learning outcomes

After completing the worksheet you should be able to:

- demonstrate knowledge and understanding of percentage errors and uncertainties
- evaluate absolute and percentage uncertainties
- determine uncertainty when data are combined by addition, subtraction, multiplication, division, and raising to powers.

Percentage uncertainties

Introduction

When something is measured there will always be a small difference between the measured value and the true value. There are several possible reasons for uncertainty in measurements, including the difficulty of taking the measurement, the precision of the measuring instrument (for example, due to the size of the scale divisions), and the natural variation of the quantity being measured. The word ‘uncertainty’ is generally used in preference to ‘error’, because ‘error’ implies something that is wrong – mistakes in making measurements should be avoided, and are not included in the uncertainty.

A measurement of 2.8 g on a scale with divisions of 0.1 g means the value is closer to 2.8 g than 2.7 g or 2.9 g. If the measurement were exactly half-way between 2.8 g and 2.9 g you would round up and record 2.9 g, so 2.8 g is anything from 2.75 g up to, but not including, 2.85 g, and the measurement is written 2.8 (± 0.05) g. The ‘(± 0.05) g’ is called the absolute uncertainty.

The percentage uncertainty in a measured value is calculated as shown below.

$$\text{percentage uncertainty} = \frac{\text{uncertainty}}{\text{measured value}} \times 100\%$$

Worked example

Question

a The distance from **A** to **B** is carefully measured as 7.500 m, using a 10 m tape measure marked in millimetre increments.

i Deduce the absolute uncertainty in the measurement.

ii Determine the percentage uncertainty in the measurement.

b The distance from **B** to **C** is measured as 6.500 m using a stick 1 m in length with no scale divisions, in difficult conditions.

i Deduce the absolute uncertainty in the measurement.

ii Determine the percentage uncertainty in the measurement.

c Calculate the absolute uncertainty in the total distance from **A** to **B** to **C**.

d Calculate the percentage uncertainty in the total distance from **A** to **B** to **C**.

Answer

a i *Step 1*

Consider the start point (**A**) and end point (**B**) of the measurement, the scale division size, and the difficulty of measuring. There will be an uncertainty in the measurement both at the start point (**A**) and the end point (**B**).

The uncertainty in the measurement = 2 mm
(If you measure a shorter length with a 30 cm ruler, there would be an uncertainty of 0.5 mm at each end, resulting in a 1 mm uncertainty overall.)

Step 2

Write out the measurement with its absolute uncertainty. The uncertainty has the same unit as the measurement.

The distance **AB** is 7.500 (± 0.002) m.

ii *Step 3*

Calculate the percentage uncertainty using the equation:

$$\text{percentage uncertainty} = \frac{\text{uncertainty}}{\text{measured value}} \times 100\%$$

$$\text{percentage uncertainty} = \frac{0.002}{7.500} \times 100\% = 0.03\%$$

b i *Step 4*

Consider the start point (**B**) and end point (**C**) of the measurement, the scale division size, and the difficulty of measuring. There will be an uncertainty in the measurement at the start point (**B**) and at the end point (**C**). Because the metre stick has no scale divisions, you can only estimate to the nearest half a metre.

The uncertainty in the measurement = 0.5 m

Step 5

Write out the measurement with its absolute uncertainty.

The distance **BC** is 6.5 (± 0.5) m.

Step 6

$$\text{The percentage uncertainty} = \frac{0.5}{6.5} \times 100\%$$

$$= 7.69\%$$

$$= 8\% \text{ (to nearest \%)}$$

Unless the percentage uncertainty is less than 1%, it is acceptable to quote percentage uncertainties to the nearest whole number.

c *Step 7*

For the distance **ABC** the two measurements are added. The overall absolute uncertainty will be the sum of the individual absolute uncertainties.

uncertainty in **ABC** = uncertainty in **AB** + uncertainty in **BC**

$$= 0.002\text{ m} + 0.5\text{ m}$$

$$= 0.5\text{ m (since 0.002 is insignificant compared to 0.5)}$$

d *Step 8*

To find the percentage uncertainty, first calculate the measured value of **ABC**.

ABC = 7.5 + 6.5 = 14.0 m

Step 9

Calculate the percentage uncertainty using the equation:

$$\text{percentage uncertainty} = \frac{\text{uncertainty}}{\text{calculated value}} \times 100\%$$

$$\text{percentage uncertainty} = \frac{0.5}{14.0} \times 100\%$$

$$= 3.57\%$$

$$= 4\% \text{ (to nearest \%)}$$

Questions

1 Write down these measurements with their absolute uncertainty.

a 6.0 cm length measured with a ruler marked in mm (*1 mark*)

b 0.642 mm diameter measured with a digital micrometer (*1 mark*)

c 36.9 °C temperature measured with a thermometer which has a quoted accuracy of: '± 0.1 °C (34 to 42 °C), rest of range ± 0.2 °C'. (*1 mark*)

2 Calculate the percentage uncertainty in these measurements.

- **a** 5.7 ± 0.1 cm (*1 mark*)
- **b** 2.0 ± 0.1 A (*1 mark*)
- **c** 450 ± 2 kg (*1 mark*)
- **d** 10.60 ± 0.05 s (*1 mark*)
- **e** 47.5 ± 0.5 mV (*1 mark*)
- **f** 366 000 ± 1000 J (*1 mark*)

3 Calculate the absolute uncertainty in these measurements.

- **a** 1200 W ± 10% (*1 mark*)
- **b** 34.1 m ± 1% (*1 mark*)
- **c** 330 000 Ω ± 0.5% (*1 mark*)
- **d** 0.008 00 m ± 1% (*1 mark*)

4 Calculate the absolute and percentage uncertainty in the total mass of suitcases of masses x, y, and z.

$x = 23.3\ (\pm\ 0.1)$ kg, $y = 18\ (\pm\ 1)$ kg, $z = 14.7\ (\pm\ 0.5)$ kg (*2 marks*)

Combining uncertainties

Introduction

In a calculation, if several of the quantities have uncertainties then these will all contribute to the uncertainty in the answer. The following rules will help you calculate the uncertainty in your final answers.

- When quantities are added, the uncertainty is the sum of the *absolute* uncertainties.
- When quantities are subtracted, the uncertainty is also the sum of the *absolute* uncertainties.
- When quantities are multiplied, the *total percentage* uncertainty is the sum of the *percentage* uncertainties.
- When quantities are divided, the *total percentage* uncertainty is also the sum of the *percentage* uncertainties.
- When a quantity is raised to the power n, the *total percentage* uncertainty is n multiplied by the *percentage* uncertainty – for example, for a quantity x^2, total percentage uncertainty = 2 × percentage uncertainty in x.

Worked example

Question

A current of 2.8 (± 0.1) A passes through a kettle element. The mains power supply is 230 (± 12) V.

Calculate the power transferred, including its uncertainty.

Answer

Step 1

Calculate the power.

$P = IV$

$P = (2.8\text{ A}) \times (230\text{ V}) = 644\text{ W}$

Step 2

Calculate the percentage uncertainties.

The percentage uncertainty in current $= \dfrac{0.1}{2.8} \times 100\% = 3.57\%$

The percentage uncertainty in voltage $= \dfrac{12}{230} \times 100\% = 5.22\%$

The percentage uncertainty in power $= 3.57\% + 5.22\% = 8.79\% = 9\%$ (to nearest %)

Step 3

Calculate the absolute uncertainty in the power.

The absolute uncertainty $= \dfrac{9}{100} \times 644\text{ W} = 58.0\text{ W}$

Step 4

State the answer with units.

Power = 644 (± 58) W

Questions

5 A piece of string 1.000 (± 0.002) m is cut from a ball of string of length 100.000 (± 0.002) m. Calculate the length of the remaining string and the uncertainty in this length. (*2 marks*)

6 A runner completes 100 (± 0.02) m in 18.6 (± 0.2) s. Calculate his average speed and the uncertainty in this value. (*2 marks*)

7 A car accelerates, with constant acceleration, from 24 (± 1) m s^{-1} to 31(± 2) m s^{-1} in 9.5 (± 0.1) s. Calculate the acceleration. State your answer with its absolute uncertainty. (*3 marks*)

8 A cube has a mass of 7.870 (± 0.001) kg and sides of length 10.0 (± 0.1) cm. Give the value of the density of the cube. (*2 marks*)

9 In a Young's slits experiment, two slits that are very close together are illuminated, and on a distant screen an interference pattern of light and dark fringes is seen. The separation of the fringes can be used to calculate the wavelength of the light. In a demonstration of this experiment:

- the double slit separation, $a = 0.20\ (\pm\ 0.01)$ mm
- the distance from the slits to the screen, $D = 4.07\ (\pm\ 0.01)$ m
- the distance between two adjacent bright fringes $x = 12.0\ (\pm\ 0.05)$ mm.

The equation for calculating wavelength is $\lambda = \frac{ax}{D}$.

a Calculate:

i the wavelength, λ, of the light (*1 mark*)

ii the absolute uncertainty in the wavelength. (*2 marks*)

b The distance between 11 fringes (10 spaces) = 120.0 (± 0.05) mm. Using this value, calculate the new absolute uncertainty in the wavelength. (*2 marks*)

c Comment on whether the uncertainty in the wavelength could be significantly reduced by increasing the number of fringes measured to, for example, 20 or more. (*1 mark*)

Maths skills links to other areas

You may also need to calculate uncertainties when considering precision and accuracy of measurements and data, including margins of error, percentage errors, and uncertainties in apparatus.